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SHORT COMMUNICATION

ON-LINE IDENTIFICATION OF INTERACTING TWO-TANK SYSTEM

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Abstract – In order to demonstrate the effectiveness of the process identification algorithm, on-line parameter estimator is evaluated experimentally by using two-tank system with interaction. On-line parameter estimator used in this paper is based on a recursive parameter estimation algorithm. MIMO linear, bilinear and quadratic models based on ARMA model are used to identify two-tank system. A quadratic model for two-tank system with interaction is developed to confirm the propriety of MIMO quadratic model used in identification of two-tank system. The results of on-line identification experiments on the two-tank system show that the estimated parameters of each model converge and the output tracking errors are bounded by disturbance bound. But, the quadratic model showed the best convergence.

Key words: On-line Identification, Two-tank System, Quadratic Model, ARMA Model

INTRODUCTION

Control of liquid inventory such as a surge tank system in a chemical plant is an important and basic problem. The traditional approach to this liquid level control problems has involved the use of controllers with propotional and integral modes. Recently, much research in this fields has been carried out [MacDonald and McAVoy, 1986; Campo and Morari, 1989]. To get an improved control strategy, we have to obtain a better knowledge about the process. But, in general, a complete description of the real physical system is almost impossible. Thus, identification of chemical processes is an important task for satisfactory control and operation of the processes. The purpose of the identification is to determine a model which is equivalent to the actual process based on past input and output data. From a prior knowledge of the process we can choose an appropriate structure of a model. The identification problem consists of the determination of unknown parameters appearing in the model of the process.

The problem of system identification has received much more attention in recent years. Many recursive identification methods have been proposed for linear model [Landau, 1976; Ioannou and Johnson, 1983; Ahmed, 1984; Solbrand et al., 1985; Delopolos and Giannakis, 1994; Schoukens et al., 1994]. These methods can be classified into two methods: the equation error method and the output error method. In the equation error method the past input and output data of the actual process are used, while in the output error method the past data of the predetermined model are used. Identification for bilinear models has been studied by Frick and Valavi [1978], Kubrusly [1981], Hwang and Chen [1985], Inagaki and Mochizuki [1984], Dorissen [1990] and King et al. [1990]. Yeo and Williams [1986] had used autoregressive moving average (ARMA) model for the identification of single variable bilinear systems.

In this study multi-input multi-output (MIMO) linear, bilinear and quadratic model based on ARMA model will be used in the identification of well-known two-tank liquid level system with interaction. One of the appealing characteristics of the bilinear and quadratic ARMA models is that they are linear in their parameters, and an identification algorithm developed for a linear ARMA model can also be used in the bilinear and quadratic ARMA models. An on-line parameter estimator based on a recursive parameter estimation algorithm is used to estimate parameters of each model.

EXPERIMENTAL SYSTEM

Fig. 1 describes a schematic diagram of a well-known two-tank level system whose liquid levels interact. The two-tank system shown in Fig. 1 is similar to the double-tank system for pH control. The level of tank 1 depends on the level of tank 2 (and vice versa) as a result of the interconnecting stream with flow rate q_1. The IBM PC (486 DX) is connected to ADA2110 analog/digital and digital/analog I/O expansion board (from Real Time Devices, Inc.). This I/O expansion board provides 16

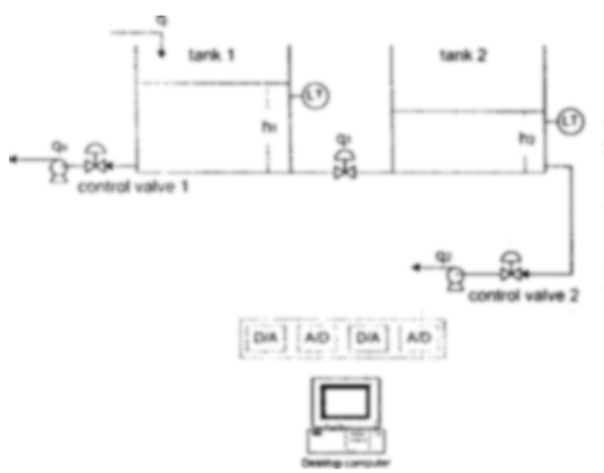

Fig. 1. Schematic diagram of two-tanks system.

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compatible digital I/O lines which can be interfaced with external devices. The inlet flow rate of tank 1 (q_i) is the manipulated disturbance which is manually operated. The outlet flow rates of tank 1 and 2 (q_o and q_2) are adjusted by a pneumatic control valve (from Badger Meter, Inc.) in proportion to its inlet air pressure which is regulated by T5200 pneumatic transducer (from FAIRCHILD Corp.). The level of tanks is measured by a 843 DP d/p Cell Transmitter (from Foxboro Corp.) which generates a 4-20 mA signal in proportional to liquid level. Both the two tanks are made of acrylic plastic columns whose diameters are 50 mm respectively.

DEVELOPMENT OF QUADRATIC MODEL FOR TWO-TANK LEVEL SYSTEM

Considering the two-tank system shown in Fig. 1. The interaction between the tanks is clearly shown from the valve Eq. (1) for flow, $q_1(t)$.

$$q_1(t)=C_V'[h_1(t)-h_2(t)]^{1/2} \tag{1}$$

Starting with an unsteady state mass balances around the two tanks we can set up the followings relations:

$$\rho q_i(t)-\rho q_1(t)-\rho q_o(t)=\rho A\frac{dh_1(t)}{dt} \tag{2}$$

$$\rho q_1(t)-\rho q_2(t)=\rho A\frac{dh_2(t)}{dt} \tag{3}$$

In order to develop the quadratic model for two-tank system with interaction, approximation around the steady state values of liquid levels (h_{1s}, h_{2s}) is carried out for Eq. (1). Using Taylor's series expansion to second order term, we have

$$\begin{aligned} q_1(t)=&(a_1-a_2h_{1s}+a_2h_{2s}+a_3h_{1s}^2+a_3h_{2s}^2-2a_3h_{1s}h_{2s})\\ &+(a_2-2a_3h_{1s}+2a_3h_{2s})h_1(t)+(2a_3h_{1s}-2a_3h_{2s}-a_2)h_2(t)\\ &+a_3h_1(t)^2+a_3h_2(t)^2-2a_3h_1(t)h_2(t) \end{aligned} \tag{4}$$

where

$$a_1=C_V'(h_{1s}-h_{2s})^{1/2}$$

$$a_2=\frac{1}{2}C_V'(h_{1s}-h_{2s})^{-1/2}$$

$$a_3=-\frac{1}{8}C_V'(h_{1s}-h_{2s})^{-3/2}$$

Substitution of Eq. (4) into Eqs. (2), (3) and rearranging yields the following quadratic model for two tanks system:

$$\begin{aligned}\frac{dh_1(t)}{dt}=&b_1q_i(t)-b_1q_o(t)-b_2h_1(t)+b_2h_2(t)-b_3h_1(t)^2\\ &-b_3h_2(t)^2+2b_3h_1(t)h_2(t)-b_4\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{dh_2(t)}{dt}=&b_2h_1(t)-b_2h_2(t)+b_3h_1(t)^2+b_3h_2(t)^2\\ &-2b_3h_1(t)h_2(t)-b_1q_2(t)+b_4\end{aligned} \tag{6}$$

where

$$b_1=1/A$$

$$b_2=(a_2-2a_3h_{1s}+2a_3h_{2s})/A$$

$$b_3=a_3/A$$

$$b_4=(a_1-a_2h_{1s}+a_2h_{2s}+a_3h_{1s}^2+a_3h_{2s}^2-2a_3h_{1s}h_{2s})/A$$

IDENTIFICATION ALGORITHM

A single variable discrete time system can be described by ARMA representation of a form

$$y(k)=p^Tx(k-1)+d(k) \tag{7}$$

where p, x(k−1), and d(k) denote the parameter vector, the past input and output data vector and the bounded disturbance respectively. The model of the system (7) can be written as

$$y^*(k|k)=p^{*T}(k)x(k-1) \tag{8}$$

$$y^*(k|k-1)=p^{*T}(k-1)x(k-1) \tag{9}$$

where $p^*(k)$ denotes the model parameter vector to be adjusted. In order to identify the system parameter vector p, we propose a recursive identification algorithm of the form

$$p^*(k)=p^*(k-1)+\xi(k-1)x(k-1)e^*(k) \tag{10}$$

where $e^*(k)$ is the prior output tracking error given by

$$e^*(k)=y(k)-y^*(k|k-1) \tag{11}$$

and the gain $\xi(k-1)$ is calculated as follows:

$$\xi(k-1)=\begin{cases}\dfrac{2\lambda(k)[\zeta(k)-1]}{\zeta(k)\|x(k-1)\|^2+\theta(k)} & ;\ \zeta(k)>1\\ 0 & ;\ \zeta(k)\le 1\end{cases} \tag{12}$$

where

$$\zeta(k)=\frac{|e^*(k)|}{qD}$$

$$0<\lambda(k)\le 1$$

$$0<\theta(k)<R_1<\infty$$

$$1\le q<R_2<\infty$$

For the multivariable discrete time system, the above identification algorithm was used with slight modification.

EXPERIMENTS AND RESULTS

Using the postulated process models of Eqs. (5) and (6), two-tank system is described by a discrete quadratic model based on ARMA model of the following form:

$$\begin{aligned}\mathbf{Y}^*(\mathbf{k})=\sum_{i=1}^{N}[&\mathbf{A}_i^*\mathbf{Y}(\mathbf{k-i})+\sum_{j=1}^{n}\mathbf{B}_{ij}^*\mathbf{Y}(\mathbf{k-i})y_j(k-i)\\ &+\mathbf{C}_i^*\mathbf{U}(\mathbf{k-i-T})]\end{aligned} \tag{13}$$

In the present study, we also used Eqs. (14) and (15) as linear and bilinear model, respectively.

$$\mathbf{Y}^*(\mathbf{k})=\sum_{i=1}^{N}[\mathbf{A}_i^*\mathbf{Y}(\mathbf{k-i})+\mathbf{C}_i^*\mathbf{U}(\mathbf{k-i-T})] \tag{14}$$

$$\begin{aligned}\mathbf{Y}^*(\mathbf{k})=\sum_{i=1}^{N}[&\mathbf{A}_i^*\mathbf{Y}(\mathbf{k-i})+\sum_{j=1}^{m}\mathbf{B}_{ij}^*\mathbf{Y}(\mathbf{k-i})u_j(k-i-T)\\ &+\mathbf{C}_i^*\mathbf{U}(\mathbf{k-i-T})]\end{aligned} \tag{15}$$

In Eqs. (13), (14) and (15) the parameter matrices are unknown and have to be estimated. The unknown parameter matrices are estimated by recursive parameter estimation method. The two-tank system can be described by second-order dynamic system. However, as the model order of system increases, the number of the estimated parameters increases as well, and so the convergence problem becomes increasingly difficult. To avoid this problem, the first-order bilinear and quadratic models were used in this study, while in the linear model the second-model order was used. One sampling time (i.e., 3 sec) was chosen as the time delay of model. This turned out to be sufficient value that reflected the time delay of the two-tank system used in this study. In the identification, the algorithm given by Eqs. (10), (11) and (12) with q=2, $\theta(k)=1$, and $\lambda(k)=\zeta(k)/2[\zeta(k)-1]$ was used. The inlet flow rate of tank 1 (q_i) was manually operated in the range of 0 to 15 ml/sec to produce the disturbance effects.

The accurate on-line measurement of the outlet flow rates of tank 1 and 2 (q_0 and q_2) was difficult because of the effect of electrical noise. Thus, the openning rates of control valves were used as input variables of model. The openning rates of control valves were controlled by pseudo-random binary sequences (PRBS) and the initial values of the model parameters were set to 0.5 and sampling time was set to 3 sec. The input signals are shown in Fig. 2. Fig. 3, 4 and 5 show the output tracking errors of each model for on-line parameter estimation. The maximum disturbance value produced by the inlet flow rate of tank 1 is 20 mm.

From these results we can see that the output tracking errors of each model are confined within the expected bound of disturbances. The convergences of the estimated parameters of each model are shown in Fig. 6. The identification results of linear, bilinear and quadratic model for two-tanks system are given in Eqs. (16), (17) and (18), respectively. From this ex-

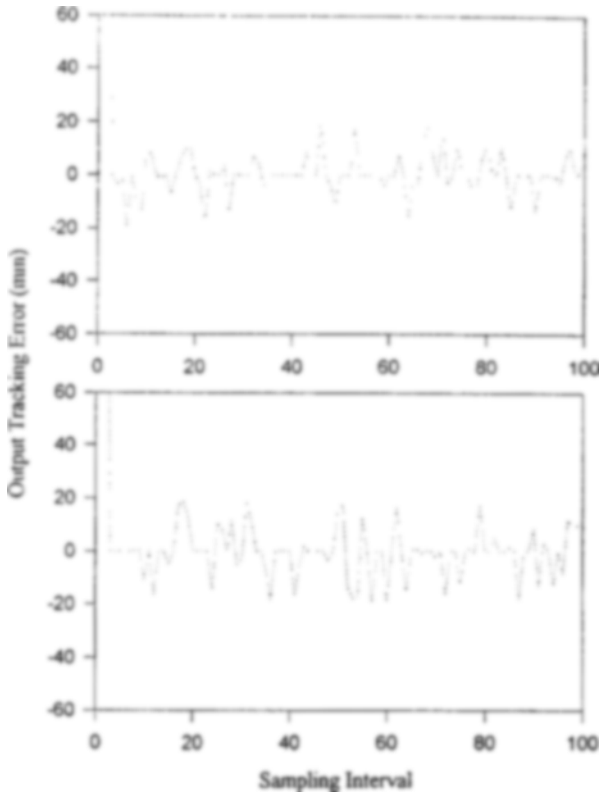

Fig. 3. Output tracking error using linear model.

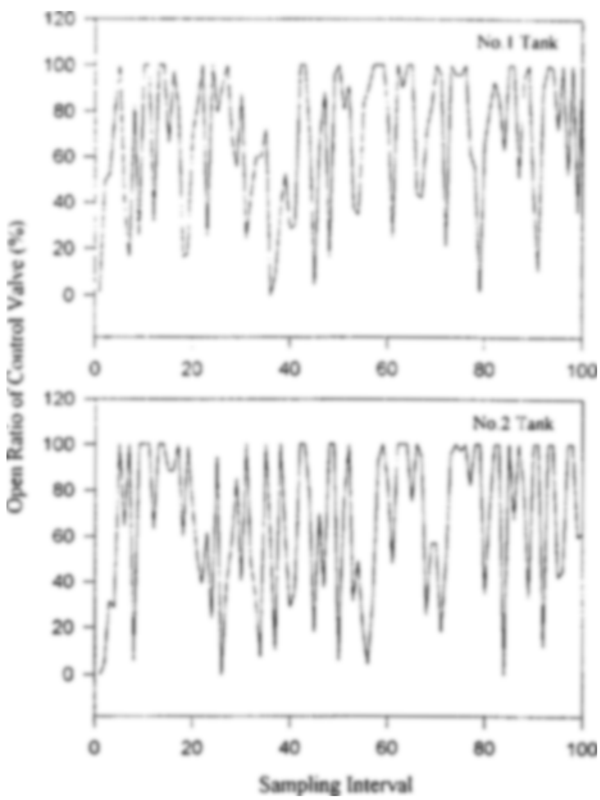

Fig. 2. Open ratio of control valve by PRBS.

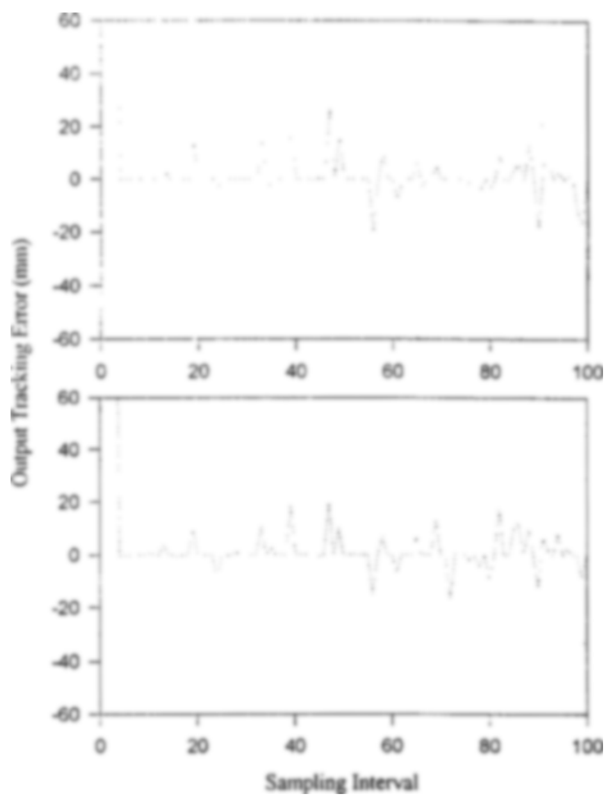

Fig. 4. Output tracking error using bilinear model.

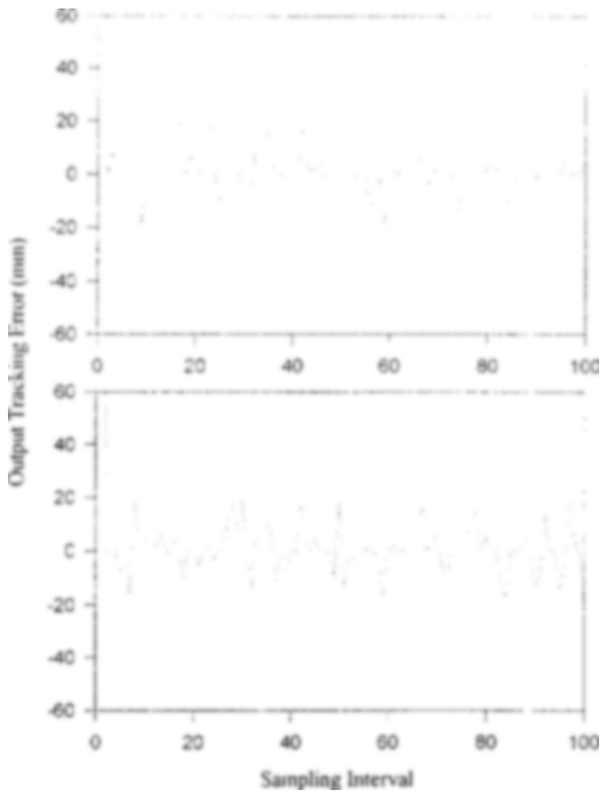

Fig. 5. Output tracking error using quadratic model.

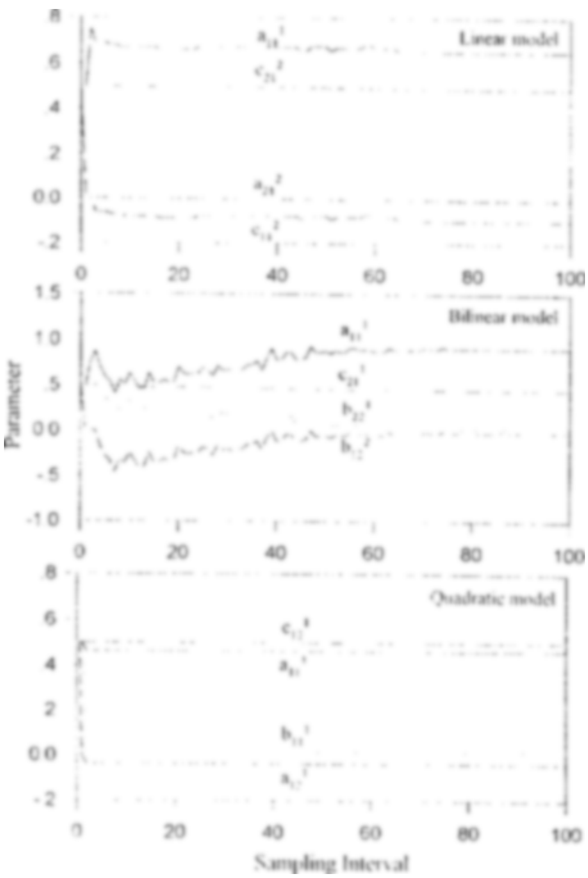

Fig. 6. On-line parameter estimation of each model for two-tank system. a_{nm}^i, b_{nm}^i and c_{nm}^i are elements of model parameter materices A_i^*, B_{li}^* and C_i^* in Eqs. (13), (14) and (15).

periment, the identification algorithm gives accurate estimates and stable output tracking errors.

$$\mathbf{Y}^*(\mathbf{k})=\begin{bmatrix}0.635 & -0.098\\ -0.005 & 0.492\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{1})+\begin{bmatrix}0.481 & -0.096\\ -0.006 & 0.491\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{2})+\begin{bmatrix}-0.034 & 0.483\\ 0.440 & -0.058\end{bmatrix}\mathbf{U}(\mathbf{k}-\mathbf{2})+\begin{bmatrix}-0.100 & 0.649\\ 0.485 & -0.005\end{bmatrix}\mathbf{U}(\mathbf{k}-\mathbf{1}) \quad (16)$$

$$\mathbf{Y}^*(\mathbf{k})=\begin{bmatrix}0.922 & -0.015\\ -0.015 & 0.916\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{1})+\begin{bmatrix}0.020 & 0.027\\ 0.025 & 0.032\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{2})u_1(k-1)+\begin{bmatrix}-0.020 & -0.008\\ -0.022 & -0.011\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{1})u_2(k-1)+\begin{bmatrix}-0.041 & 0.455\\ 0.460 & -0.046\end{bmatrix}\mathbf{U}(\mathbf{k}-\mathbf{1}) \quad (17)$$

$$\mathbf{Y}^*(\mathbf{k})=\begin{bmatrix}0.464 & -0.036\\ -0.036 & 0.464\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{1})+\begin{bmatrix}0.021 & 0.022\\ 0.018 & 0.019\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{2})y_1(k-1)+\begin{bmatrix}0.022 & 0.023\\ 0.019 & 0.021\end{bmatrix}\mathbf{Y}(\mathbf{k}-\mathbf{1})y_2(k-1)+\begin{bmatrix}-0.000 & 0.500\\ 0.498 & -0.000\end{bmatrix}\mathbf{U}(\mathbf{k}-\mathbf{1}) \quad (18)$$

CONCLUSION

In this study, an identification problem of a well-known two-tank system was investigated using a recursive parameter estimation algorithm. In the case of bilinear and quadratic model, in spite of the incorrect model order, the identification algorithm used in this study gave accurate estimates. Among three models studied, the quadratic model showed the best convergence. But, the analysis of the convergence properties of the identification method for the quadratic model is not yet studied and is a topic for future research.

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NOMENCLATURE

A : cross-sectional area of tank
C_{v}^{i} : valve coefficient
D : disturbance bound
d : disturbance
e, e^* : control output error
h : liquid level
k : time (sampling interval)
m : the number of input variables

N : process order
n : the number of output variables
p : process parameter vector
p^* : model parameter vector
q : identification parameter
R_i : constants
T : time delay
U : plant input vector, $\in R^{m \times 1}$
u : process input
x : process data vector
Y : plant output vector, $\in R^{n \times 1}$
Y^* : model output vector, $\in R^{n \times 1}$
y : process output
y^* : model output

Greek Letters

ρ : liquid density
ζ : normalized control output error
ξ : gain of identification algorithm
λ : identification parameter
θ : identification parameter

REFERENCES

Ahmed, M. S., "Fast GLS Algorithm for Parameter Estimation", *Automatica,* **20**, 231 (1984).

Campo, P. J. and Manfred M., "Model Predictive Optimal Averaging Level Control", *AIChE J.,* **35**, 579 (1989).

Delopolos, A. and Giannakis, G. B., "Consistent Identification of Stochastic Linear System with Noisy Input-Output Data", *Automatica,* **30**, 1271 (1994).

Dorissen, H. T., "A Method for Bilinear System Identification", *IFAC Proc., Tallin,* 3 (1990).

Frick, P. A. and Valavi, A. S., "Estimation and Identification of Bilinear Systems", *Atomat. Contr. Theory and Applications,* **6**, 1 (1978).

Hwang, C. and Chen, M. Y., "Paramèter Identification of Bilinear Systems Using the Galerkin Method", *Int. J. Systems Sci.,* **16**, 641 (1985).

Inagaki, M. and Mochizuki, H., "Bilinear System Identification by Volterra Kernels Estimation", *IEEE Trans. Automat. Contr.,* **AC-29**, 746 (1984).

Ioannou, P. and Johnson, Jr., C. R., "Reduced-order Performance of Parallel and Series-Parallel Identifiers with Weakly Observable Parasitics", *Automatica,* **19**, 75 (1983).

King, P. J., Burnham, K. J. and James, D. J. G., "Comparison of Two Identification Techniques for Bilinear Systems", *Control and Comput.,* **18**, 4 (1990).

Kubrusly, C. S., "Identification of Discrete-Time Stochastic Bilinear Systems", *Int. J. Control,* **33**, 291 (1981).

Landau, I. D., "Unbiased Recursive Identification Using Model Reference Adaptive Techniques", *IEEE Trans. Automat. Contr.,* **AC-21**, 194 (1976).

MacDonald, K. A. and McAVoy, T. J., "Optimal Averaging Level Control", *AIChE J.,* **32**, 75 (1986).

Schoukens, J., Bintelon, R. and Van Hamme, H., "Identification of Linear Dynamic Systems using Piecewise Constant Excitations: Use, Misuse and Alternatives", *Automatica,* **30**, 1153 (1994).

Solbrand, G., Ahlen, A., Ljung, L., "Recursive Methods for Off-line Identification", *Int. J. Control,* **41**, 177 (1985).

Yeo, Y. K. and Williams, D. C., "An Adaptation Algorithm for Parallel Model Reference Adaptive Bilinear Systems", *IEEE Trans. Automat. Contr.,* **AC-31**, 1071 (1986).